UNSTABLE ENTROPY IN SMOOTH ERGODIC THEORY

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In this survey we study some roles of metric partial entropy along unstable lamination, a notion firstly defined and used by Ledrappier and Young [5].

In the first section we recall the definitions and the basic properties of metric and topological unstable entropies. Let f be a diffeomorphism and F an expanding (or non-uniformly as in Ledrappier-Young context) expanding invariant foliation and ξ a measurable incresing partition sub-ordinated to F, then

$$h\ (f) = H\ (f^{-1}\xi j\xi).$$

Although unstable leaves are non-compact, one can define topological entropy as usual and prove a variational principle which shows that the topological entropy is the supremum over all metric entropies along unstable foliation (work by Hu, Hua and Wu [3]). The notion of topological entropy along unstable foliation coincides with the volume growth, firstly defined by Newhouse [9] and Yomdin [14] and later developped by Saghin and Xia [10]:

$$\chi_u(f) = \sup_{x \in M} \chi_u(x, \delta)$$

where

$$\chi_u(x, \delta) := \limsup_{n \to \infty} \frac{1}{n} \log vol(f^n(W^u(x, \delta))).$$

In the second section we recall the Margulis [7] construction of Bowen-Margulis measures of maximal entropy in the uniformly hyperbolic setting and its relation to the unstable entropy and pose some open questions on the existence of invariant measures which maximize unstable entropy in partially hyperbolic context.

In the third section, we review a joint work with J. Yang [12] which shows that unstable entropy is a key notion for the so called invariance principle proved by Avila-Viana [1] and give a new criterion for invariance principle. We recall a Furstenberg [2] result for product of random matrices and its generalizations by Ledrappier [4] (linear cocycles) and later by Avila-Viana (non-linear cocycles) called an invariance principle.

The criterion in [12] had been usefull to prove some results on stability of non-uniform hyperbolicity ([6]) and also to show that the Bochi-Mañé theorem is false, in general, for linear cocycles over non-invertible maps (Viana-Yang [13]): there are C^0 open subsets of linear cocycles that are not uniformly hyperbolic and yet have Lyapunov exponents bounded from zero.

Finally we discuss the role of unstable entropy in the proof of rigidity results for Anosov diffeomorphisms on $\mathbb{T}^3$ as proved in [11] and [8].

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