Boundary of branching random walks on hyperbolic groups

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Abstract

Let Γ be a nonamenable finitely generated infinite hyperbolic group with a symmetric generating set S, and $\partial\Gamma$ the hyperbolic boundary of its Cayley graph. Fix a symmetric probability μ on Γ whose support is S, and denote by $\rho = \rho(\mu)$ the spectral radius of the random walk ξ on Γ associated to μ. Let ν be a probability on $\{1, 2, 3, \cdots\}$ with a finite mean λ. Write $\Lambda \subseteq \partial\Gamma$ for the boundary of the branching random walk with offspring distribution ν and underlying random walk ξ, and $h(\nu)$ for the Hausdorff dimension of Λ. When $\lambda > 1/\rho$, the branching random walk is recurrent, trivially

$$\Lambda = \partial\Gamma, \ h(\nu) = \dim(\partial\Gamma).$$

In this talk, we focus on the transient setting i.e. $\lambda \in [1, 1/\rho]$, and prove the following results: $h(\nu)$ is a deterministic function of λ and thus denote it by $h(\lambda)$; and $h(\lambda)$ is continuous and strictly increasing in $\lambda \in [1, 1/\rho]$ and $h(1/\rho) \leq \frac{1}{2}\dim(\partial\Gamma)$; and there is a positive constant C such that

$$h(1/\rho) - h(\lambda) \sim C\sqrt{1/\rho - \lambda} \text{ as } \lambda \uparrow 1/\rho.$$

The above results confirm a conjecture of S. Lalley in his ICM 2006 Lecture (the critical exponent of Hausdorff dimensions of boundaries of branching random walks on hyperbolic groups is 1/2).

This talk is based on a joint work with Shi Zhan, Sidoravicius Vladas and Wang Longmin.